Formations of Mobile agents with Messages Loss and Delay

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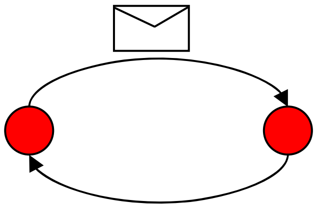
Research Goal

- Encompassing theory:

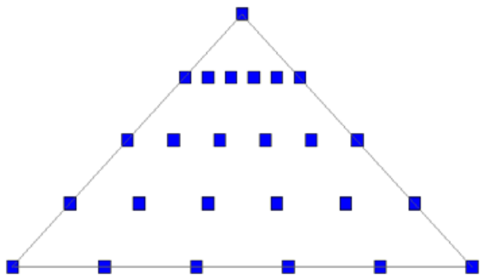
Distributed Control	Distributed Computing
• Synchronous steps • Instantaneous communication • Generally **deterministic**	• Arbitrary delay for channels and processors • Generally **non deterministic**

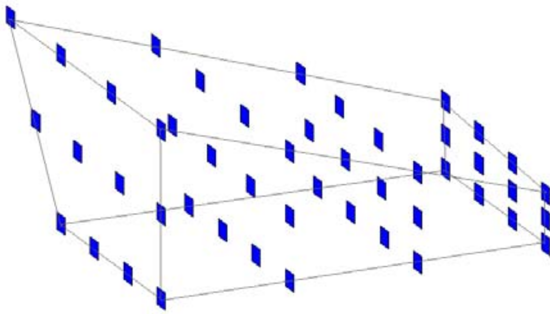
$$\frac{\partial x_j}{\partial t} = c\sum_k (x_k - x_j)$$



Main Contributions

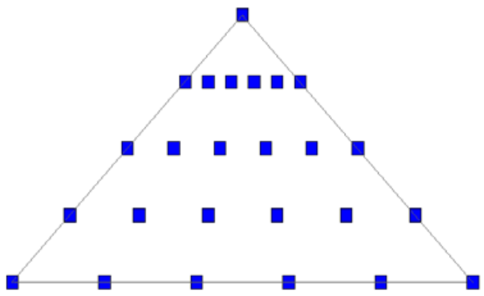
- Use of **Temporal Logic** to prove convergence in continuous time systems
- Class of Problems: **Pattern formation**
  - Agents move in Euclidean space
  - Smaller and smaller adjustments to the position but never terminate
- Communication model: **Asynchronous** message passing with loss, reordering, and delay

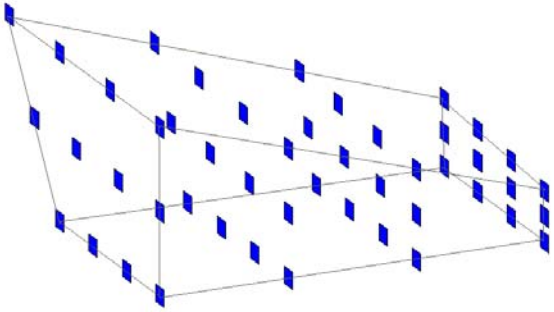




Formations of Spatial Patters

- **Distributed Control**:
  – Robots are deployed over some area
  – Apply very **simple local protocol** to update their positions **synchronously**
  – Goal: Converge to a target **global configuration**
    - Proofs of convergence: using eigen values of the transition matrix



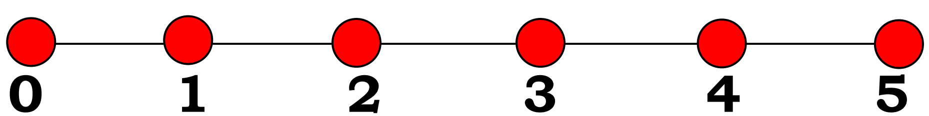


Line-up Pattern Formation

- N + 1 agents with unique indices in [0,N] and common coordinate system
- **Ideal position predicate**: Equidistance on a straight line

$$\forall x_0^*, x_N^* \in \Re, \forall i \in \{1, \ldots, N-1\}$$

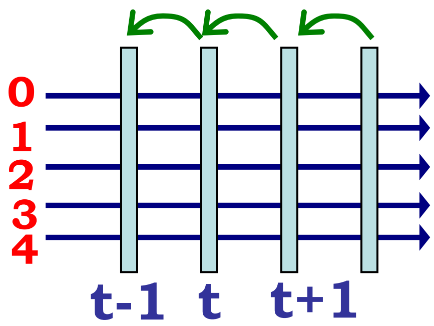
$$x_i^* = x_0^* \frac{N-i}{N} + x_N^* \frac{i}{N}$$



Line-up Synchronous Protocol

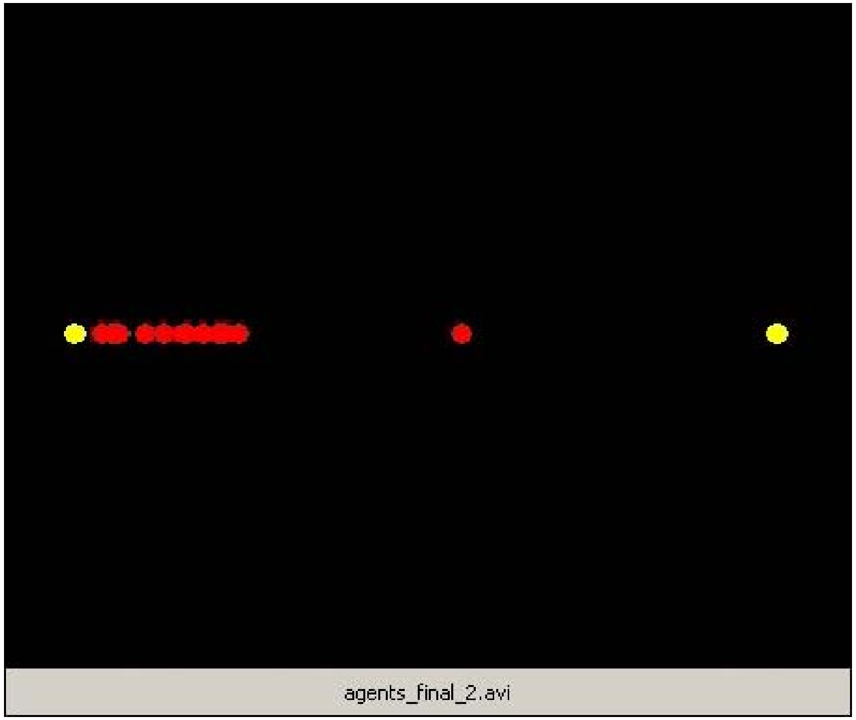
- Starting from any initial configuration
- Applying infinitely often

$$x_i(t+1) = \begin{cases} x_0 & \text{if } i = 0 \\ \dfrac{x_{i-1}(t) + x_{i+1}(t)}{2} & \text{if } 0 < i < N \\ x_N & \text{if } i = N \end{cases}$$



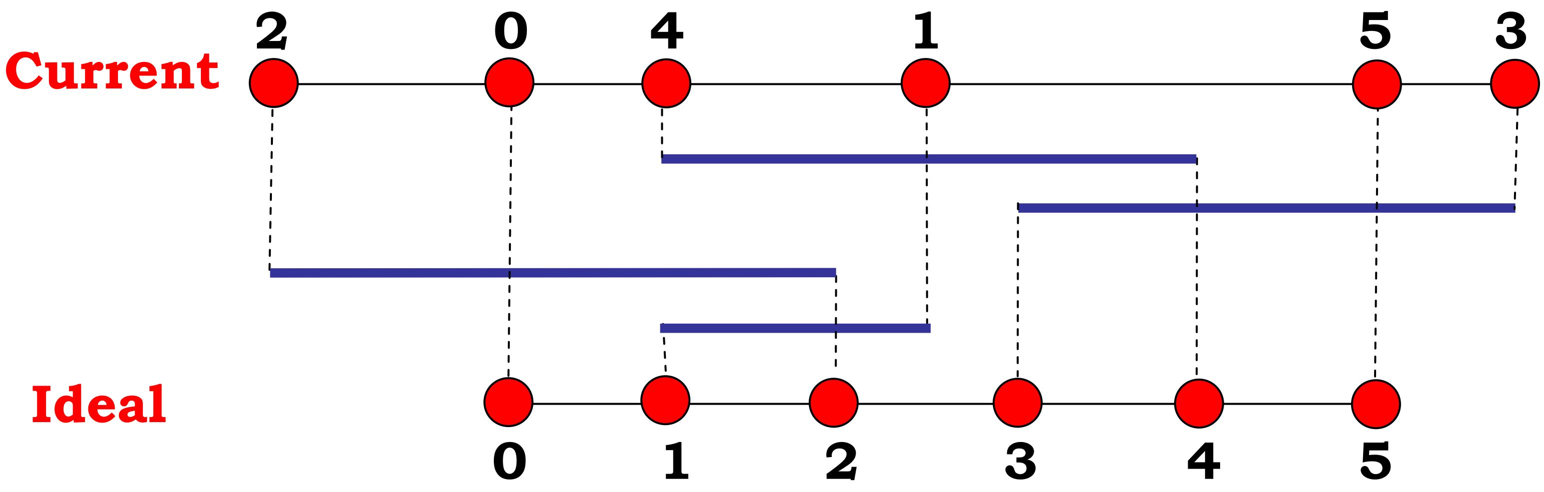
- Agents converge to the ideal positions: equidistance points on the line between $\boldsymbol{x_0}$ and $\boldsymbol{x_N}$

Line-up Protocol Simulation

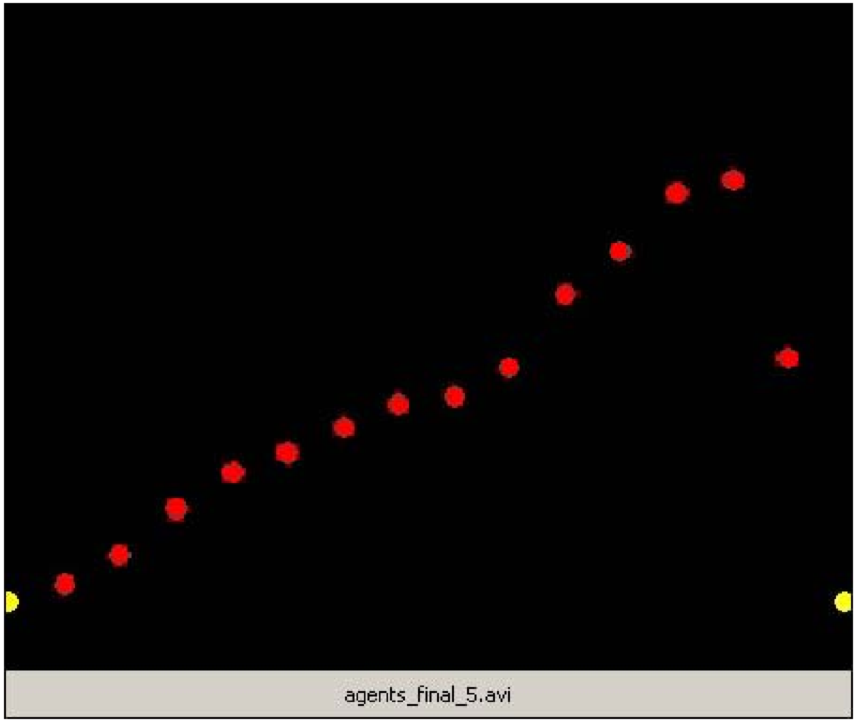


Deviation of Agents

- Distance between the agent's current location from its ideal location



Deviation Simulation

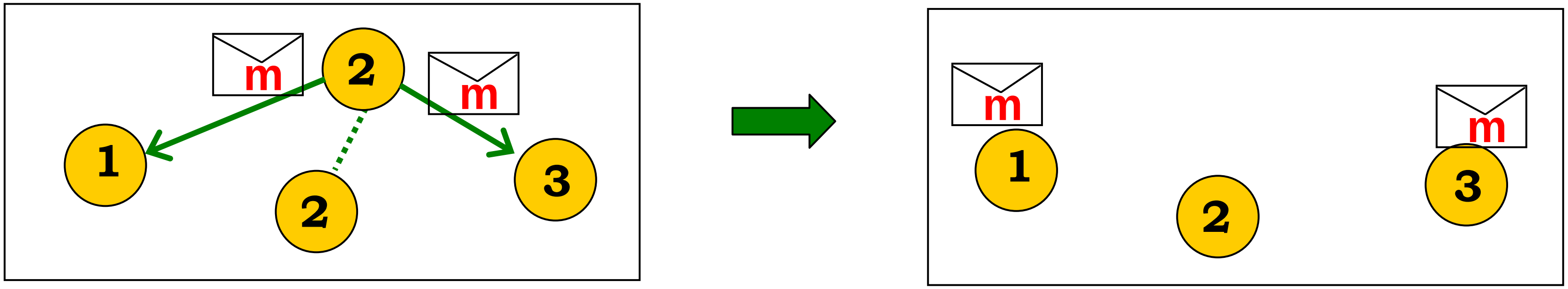


Message-Passing Systems

- Robots are not synchronized
- Robots are not able to access the state of other robots
- Realistic communication model: **Message-passing**
  - Messages may be: **lost**, **delayed** (delivered in bounded time), or **reordered**
    - Hence, messages may contain **old information**
- **Goal**: systematic way for proving convergence by combining invariance assertions with temporal logic

Asynchrony and Non-Determinism

- Asynchrony, message delays and losses lead to a non-deterministic system



- Agents 1, 3 update their states with an **old value** of the state of agent 2!
- Can collections of agents form **stable patters** when each agent updates its state using the state of other agents at some **unknown time in the past**?

Communication Model

- Agents **broadcast** their state (position) **infinitely often**
  - Communication based on indices rather than distances
- Messages:
  - can be lost or delivered in **bounded *but unknown* time**
  - Weak assumption: Delivery of some messages to agent ***i*** from at least one of the other agents

Line-Up Protocol

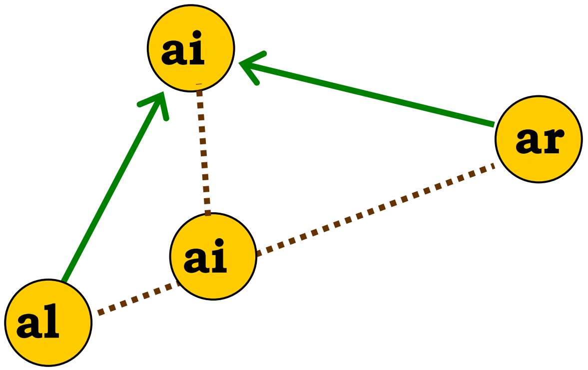
Agent ***i*** performs the following tasks:

(1) Periodically broadcasts messages containing its target position xt_i

(2) Upon receiving a message from a lower agent ***l*** and higher ***r*** computes its new target position:

$$xt_i = \frac{i-l}{r-l} xr + \frac{r-i}{r-l} xl$$

(3) Moves toward the target position xt_i



Weighted-Average of al and ar

$$xt_i = \frac{i-(i-1)}{(i+1)-(i-1)} xr + \frac{(i+1)-i}{(i+1)-(i-1)} xl$$

Deviations in Asynchronous System

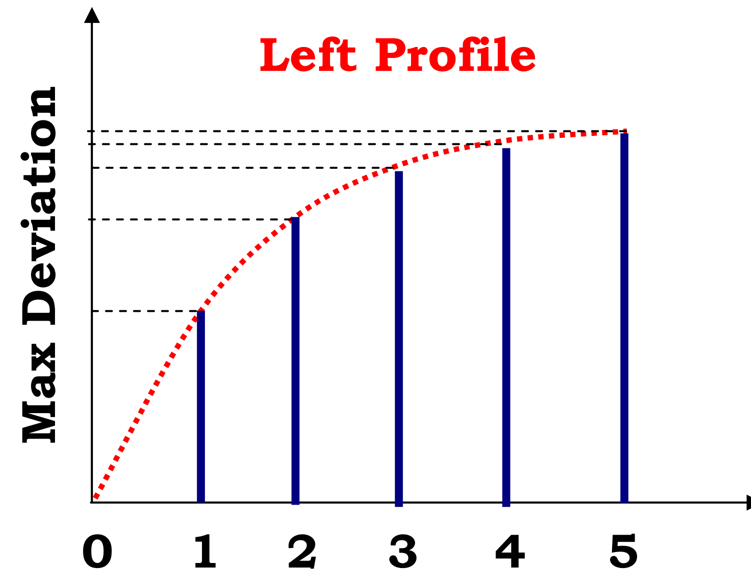
- Distance between the agent's current location from its ideal location
- <u>**Max Deviation**</u>:
  - Messages in the channels $\Rightarrow$ in the same state several positions associated with the same agent (state variables and messages in transit) $\Rightarrow$ take the max of these!

Deviation Profile

- **Left profile**: **L**(*s*)

$$\forall j \in [0, N]$$

$$\max\_dev(j, s) \le \max(s)\left(1 - \frac{1}{2^{j}}\right)$$



- **Right profile**: **R**(*s*)

$$\forall j \in [0, N]$$

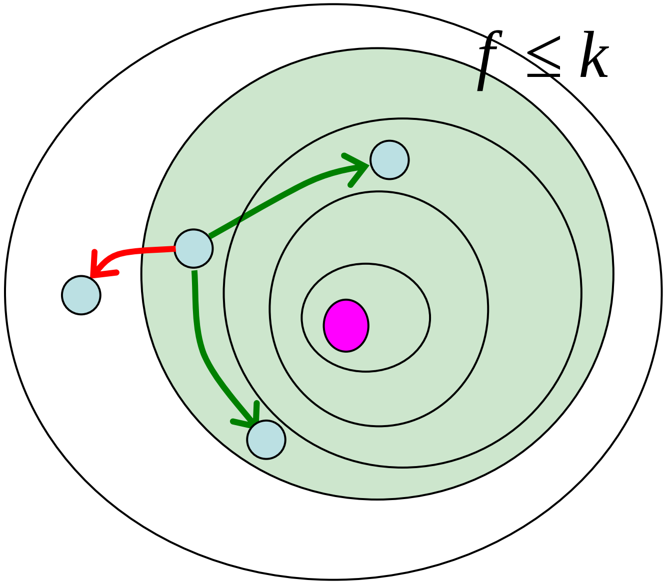
$$\max\_dev(j, s) \le \max(s)\left(1 - \frac{1}{2^{N-j}}\right)$$

Proof Technique

- **Safety**: ***f***: States → R

$$\forall k \quad stable(f \le k)$$

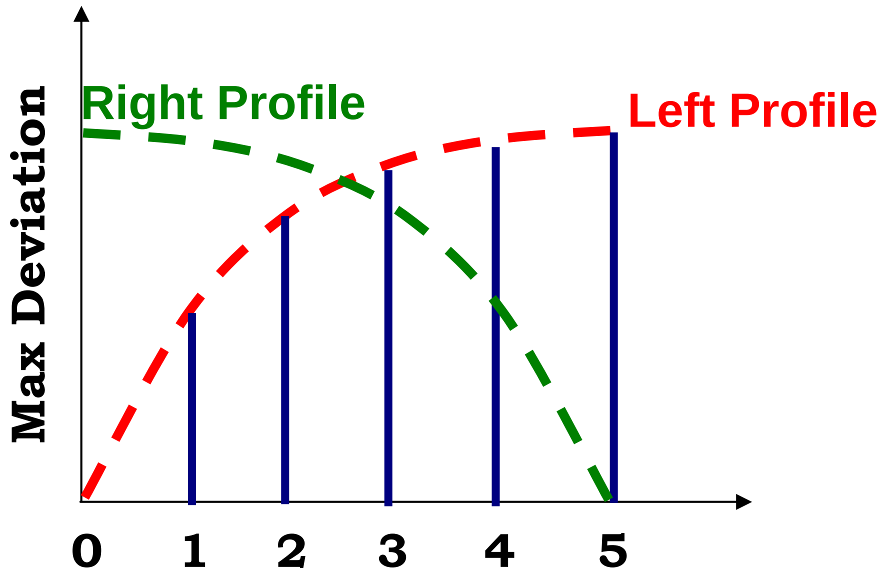
$$\forall a \quad \{f(s) \le k\} \xrightarrow{a} \{f(s') \le k\}$$

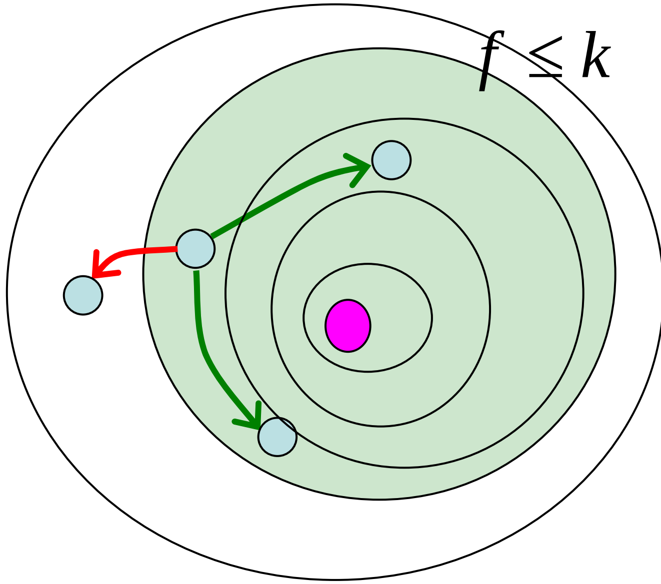


- **Progress**:

$$\exists a \quad \{f(s) \le k\} \xrightarrow{a} \{f(s') \le \alpha_k k\} \qquad 0 \le \alpha_k < 1$$

Technique for the Line-up Problem

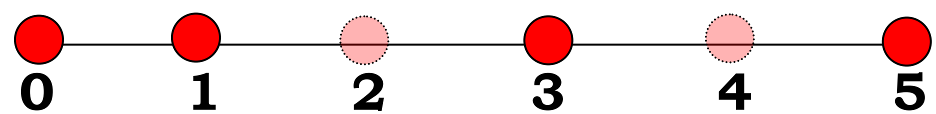




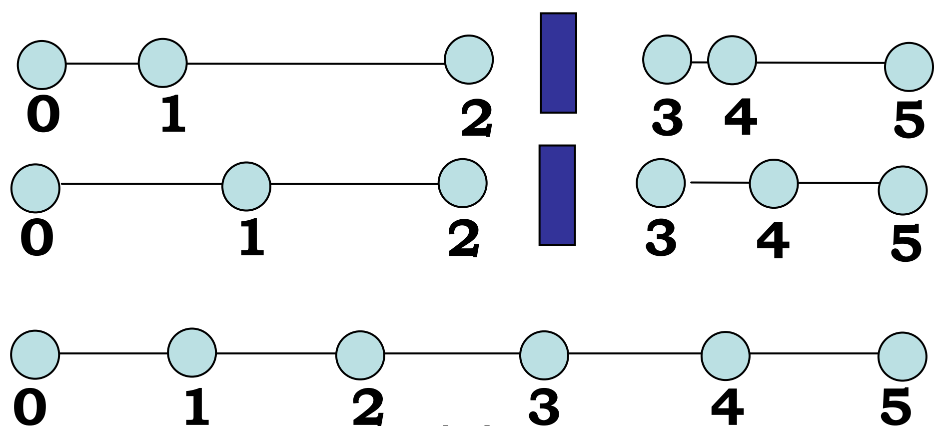
- Profile Invariant: convex combination
- Profile Decrease: eventual time-bounded delivery of messages

Advantages of the Protocol

- Agent ***i***
  - only the ***relative rank*** of ***r*** and ***l***: (***i*** - ***l***) and (***r*** – ***i***)
  - Does not know the total number of agents
- Robustness: Work even if
  - some agents fail

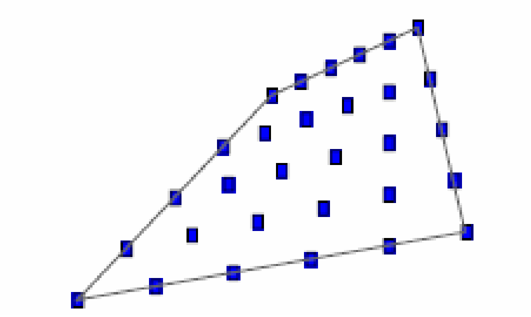


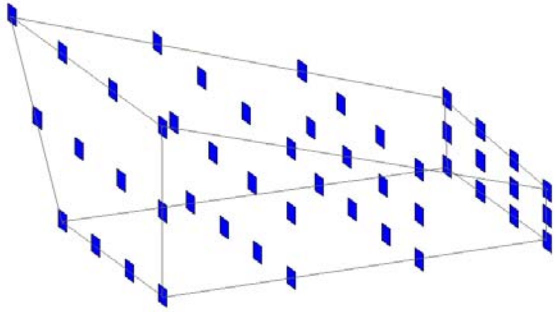
  - Set of agents is partitioned



General class of Problems

- Lines, triangles, quadrilaterals and generalization to 3 and higher dimensions
- Indices of the agents are a *d*-dimensional tuple
- Agent (***i***, ***j***) can communicate with any agent with the same ***i*** or ***j***





Conclusions and Future Work

- Took a step towards integrating distributed control and distributed computing:
  - pattern formation with mobile agents
- Future: Develop theorem-prover based framework for verifying mobile agent systems
- Model the dynamics of the motion of the agents

Line-Up Pattern Formation

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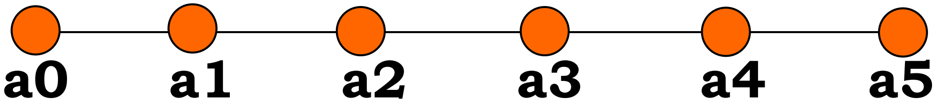
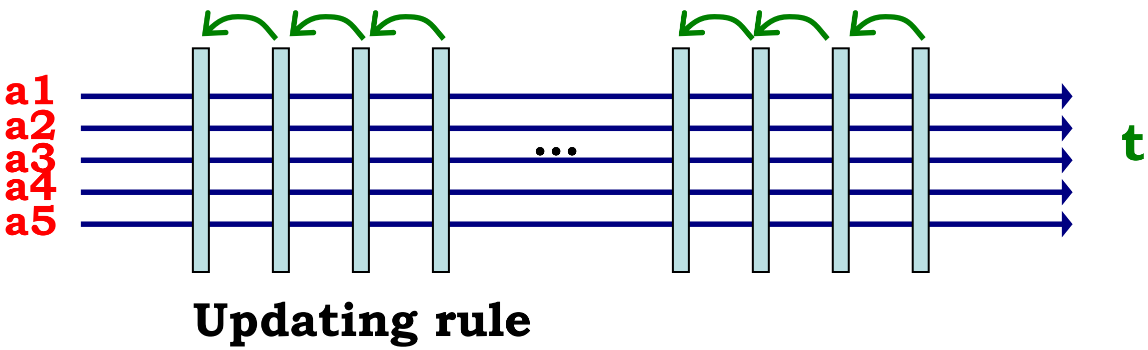


Limitations of synchronous proofs

- Convergence in Synchronous Systems



- Using eigenvalues of the transition matrix and Markov Chains

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

Communication Model

- Agents ***broadcast*** their values infinitely often
  - Communication not based on distances
- Channel: ***buffer***
- Messages:
  - can be lost or delivered in <u>bounded *but unknown*</u> time
  - Weak assumption: Delivery of some messages to agent ***i*** from at least one of the other agents