

# Formal Methods and Theorem Proving Using PVS

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# PVS: Prototype Verification System

- Specification Language integrated with a interactive Theorem Prover
- Used for writing formal specifications and checking formal proofs.
- Free Software
  - SRI International
  - Solaris, Linux and MAC
  - Implemented in LISP and interface Emacs

# Outline

- PVS
  - Specification Language
  - Prover Commands
- Distributed System example: **Local-Global Relations**
  - Theory
  - PVS Specification
  - PVS Proofs

# A PVS Example

```
sum: THEORY  
BEGIN
```

```
n:VAR nat
```

```
sum(n): RECURSIVE nat =  
( IF n = 0 THEN 0 ELSE n + sum(n-1) ENDIF )  
MEASURE n
```

```
square(n):nat = n*n
```

```
sum_of_values:
```

```
LEMMA FORALL (k:nat): sum(k) = k*(k+1) / 2
```

```
END sum
```

# A PVS Example

```
sum: THEORY
BEGIN
```

Theory Definition

```
n:VAR nat
```

```
sum(n): RECURSIVE nat =
(IF n = 0 THEN 0 ELSE n + sum(n-1) ENDIF )
MEASURE n
```

```
square(n):nat = n*n
```

```
sum_of_values:
```

```
LEMMA FORALL (k:nat): sum(k)=k*(k+1)/2
```

```
END sum
```

# Theories

- **Theory**: a collection of definitions, assumptions, axioms, and theorems.

- stored in a `.pvs` file (e.g. `sum.pvs`)

- Parametric Theories

```
sum [N0:nat]: THEORY
```

```
BEGIN
```

```
<...>
```

```
END
```

- Hierarchical Theories

```
IMPORTING sum[10]
```

# Built-in and Pre-Defined Theories

- Built-in Theories: Prelude
  - E.g. integer, Boolean, real, list, set, finite set...
  - <http://www.cs.rug.nl/~gr1/ar06/prelude.html>
  - M-x view-prelude-theory
- Pre-Defined Theories:
  - NASA Langley PVS theories
    - [shemesh.larc.nasa.gov/fm/ftp/larc/PVS-library/pvslib.html](http://shemesh.larc.nasa.gov/fm/ftp/larc/PVS-library/pvslib.html)
    - E.g. algebra, complex numbers, graphs, logarithm and exponential

# A PVS Example

```
sum: THEORY
BEGIN
```

```
n:VAR nat
```

 Variable Declaration

```
sum(n): RECURSIVE nat =
(IF n = 0 THEN 0 ELSE n + sum(n-1) ENDIF )
MEASURE n
```

```
square(n):nat = n*n
```

```
sum_of_values:
```

```
LEMMA FORALL (k:nat): sum(k) = k*(k+1)/2
```

```
END sum
```

# Variables and Constants

- Variable Declarations

- `n, m, p: VAR nat`

- Constant Declaration and Definition

- `n0: nat`

- `n0: nat = 10`

# A PVS Example

```
sum: THEORY  
BEGIN
```

```
n:VAR nat
```

```
sum(n): RECURSIVE nat =  
( IF n = 0 THEN 0 ELSE n + sum(n-1) ENDIF )  
MEASURE n
```

Function Declaration & Definition

```
square(n):nat = n*n
```

```
sum_of_values:
```

```
LEMMA FORALL (k:nat): sum(k) = k*(k+1)/2
```

```
END sum
```

# Functions

## • Declarations

- `square(n) : nat`

## • Definitions

- `square(n) : int = n*n`

- `sum(n) : RECURSIVE nat =  
( IF n=0 THEN 0 ELSE n+sum(n-1) ENDIF )  
MEASURE n`

# A PVS Example

```
sum: THEORY
BEGIN
```

```
n:VAR nat
```

```
sum(n): RECURSIVE nat =
(IF n = 0 THEN 0 ELSE n + sum(n-1) ENDIF )
MEASURE n
```

```
square(n):nat = n*n
```

```
sum_of_values:
```

```
LEMMA FORALL (k:nat): sum(k) = k*(k+1)/2
```

**Formula Declaration**

```
END sum
```

# Formulas

## ● Formula Declarations

- `sum_of_values:`

`LEMMA FORALL (k:nat) :`

`sum(k) = k * (k+1) / 2`

- Others: `CLAIM, FACT, THEOREM, PROPOSITION, ...`

## ● Expressions

- Boolean

● `=, /=, TRUE, FALSE, AND, OR, IMPLIES, IFF`

- Numeric

● `0, 1, ..., +, *, /, -, <, >, ...`

- Binding (local scope for variables)

● `FORALL, EXISTS`

# A PVS Example

```
sum: THEORY
BEGIN
```

```
n:VAR nat
```

```
sum(n):RECURSIVE nat =
(IF n = 0 THEN 0 ELSE n + sum(n-1) ENDIF )
MEASURE n
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square(n):nat = n*n
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```
sum_of_values:
```

```
LEMMA FORALL (k:nat) : sum(k) = k*(k+1)/2
```

```
END sum
```

# PVS Types

- Type System combined with higher order logic
- Base and Build-in types
  - `bool, int, real, nat, ...`
- User-defined types:
  - Keyword: `TYPE`
  - Uninterpreted Type
    - `Type1 : TYPE`
    - Defined equality predicate.
  - Interpreted Function Type
    - `Type2 : TYPE = [int, int->int]`
    - `Type3 : TYPE = FUNCTION[int, int->int]`
    - `Type4 : TYPE = ARRAY[int, int->int]`
    - `Type2, Type3, Type4` are equivalent

# Type Checking

- Undecidable
- Generate proof obligations:
  - TCCs: Type-Correctness Conditions
- Running PVS
  - Type check:  $M-x \text{ tc}$
  - Show TCCs:  $M-x \text{ show-tccs}$
  - Many of these proof obligations can be discharged automatically:  $M-x \text{ tcp}$

# TCCs of sum theory

```
%Subtype TCC generated (at line 7, column 32) for n-1
%expected type nat
%proved - complete
sum_TCC1:OBLIGATION FORALL (n:nat): NOT n=0 IMPLIES n-1>=0;
```

```
%Termination TCC generated (at line 7, column 28) for
sum(n-1)
%proved - complete
sum_TCC2:OBLIGATION FORALL (n:nat): NOT n=0 IMPLIES n-1<n;
```

```
sum(n): RECURSIVE nat =
( IF n = 0 THEN 0 ELSE n + sum(n-1) ENDIF )
MEASURE n
```

# Function Evaluation

- Execute functions

- M-x pvs-ground-evaluator
- New Buffer with the process <GndEval> ...

- Example

- <GndEval> "square(3)"

```
cpu time 0 msec user, 0 msec system
```

```
real time 0 msec
```

```
space: 3 cons cells, 0 other bytes, 0
```

```
static bytes
```

```
==> 9
```

# Using PVS

- Interactive Proof Checker
- Combine basic deductive steps and user-defined procedures
- Proofs stored in `file.prf`
- Proofs are stored as a sequence of rules
  - M-x `show-proof`
  - M-x `install-proof`
  - M-x `edit-proof`
- Maintain a Proof Tree
  - M-x `x-show-proof`

# Proofs

- Consider the lemma

```
sum_of_values:
```

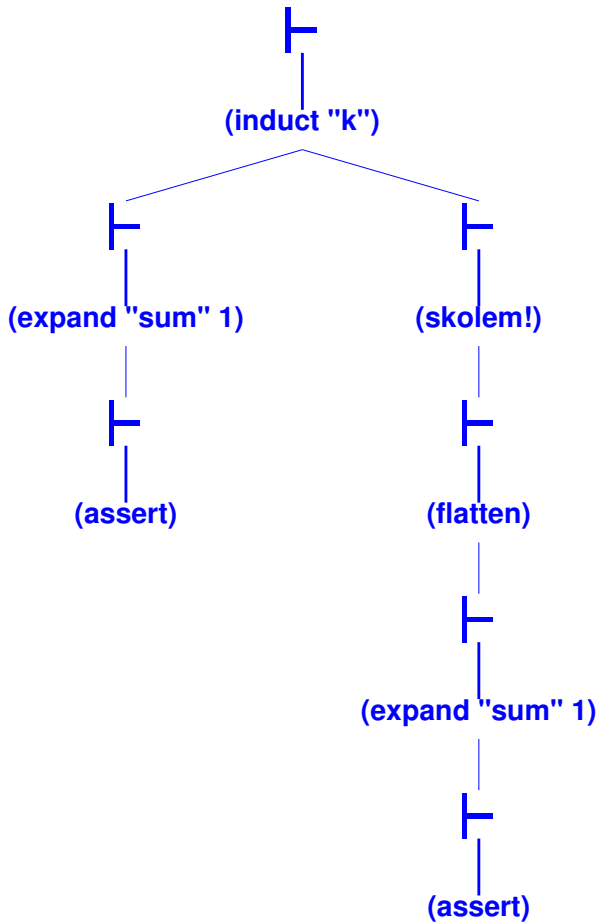
```
LEMMA FORALL (k:nat) : sum(k) = k * (k+1) / 2
```

- M-x prove, M-x pr

```
{-1} A
[-2] B
{-3} C
|-----
[1] P
{2} Q
Rule?
```

- A, B, C antecedent
- P, Q consequent
- A AND B AND C  
IMPLIES P OR Q
- [n]: formula n is unaffected by the last proof step

# Sum of Values Lemma



(M-x xpr)

sum\_of\_values :

|-----

{1} FORALL (k: nat): sum(k) = (k \* (k + 1)) / 2

Rule? **(induct "k")**

Inducting on k on formula 1, this yields 2 subgoals:

sum\_of\_values.1 :

|-----

{1} sum(0) = (0 \* (0 + 1)) / 2

Rule? **(expand "sum" 1)**

Expanding the definition of sum, this simplifies to:

sum\_of\_values.1 :

|-----

{1} 0 = 0 / 2

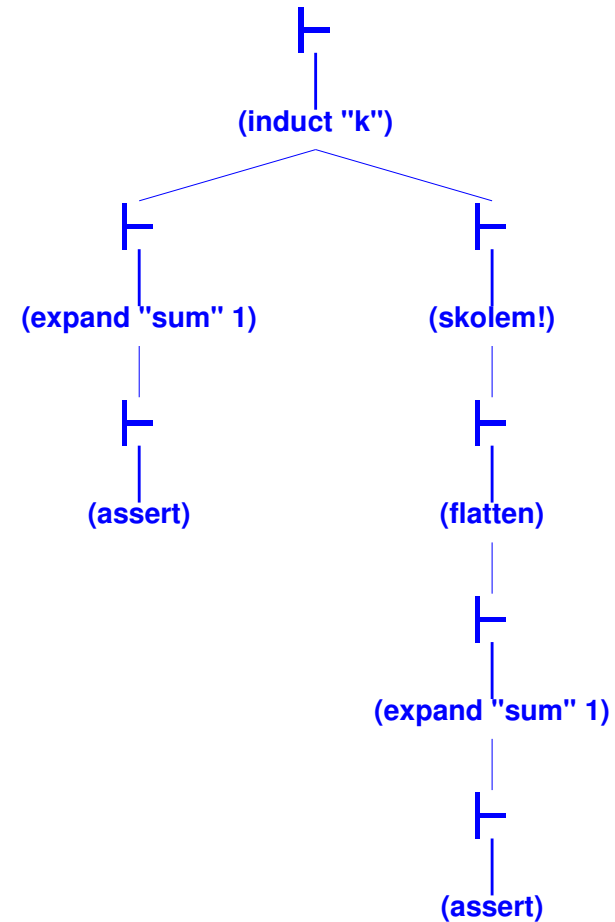
Rule? **(assert)**

Simplifying, rewriting, and recording with decision procedures,

This completes the proof of sum\_of\_values.1.

# Rule Syntax

- Surrounded by Parenthesis
- Can have arguments and change only parts of the formulas
- Examples
  - `(assert)`, `(undo)`, `(quit)`
  - `(expand "sum" +)`
  - `(expand "sum" -)`
  - `(expand "sum" (-1, -2, 2))`



# Distributed System Example

# Correctness of Distributed Systems

- Global properties to maintain

Invariants  $h(s) = h(s')$

Lyapunov  $g(s) > g(s')$

# Correctness of Distributed Systems

- Global properties to maintain

Invariants  $h(s) = h(s')$  e.g.  $avg(s) = avg(s')$

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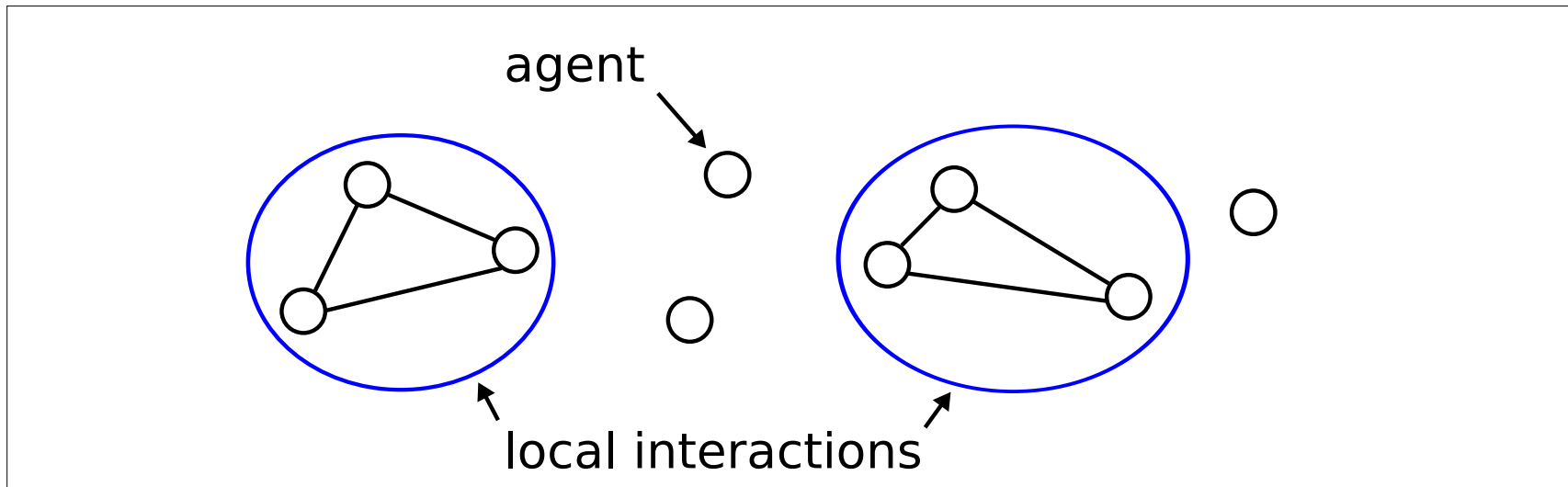
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- Local interactions



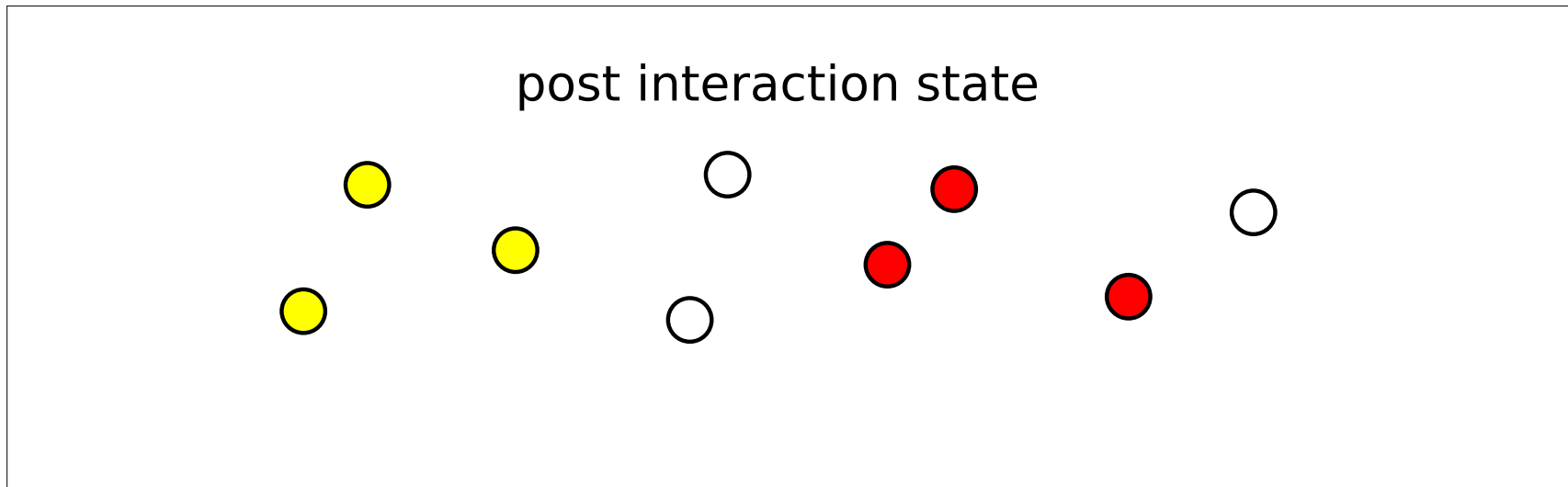
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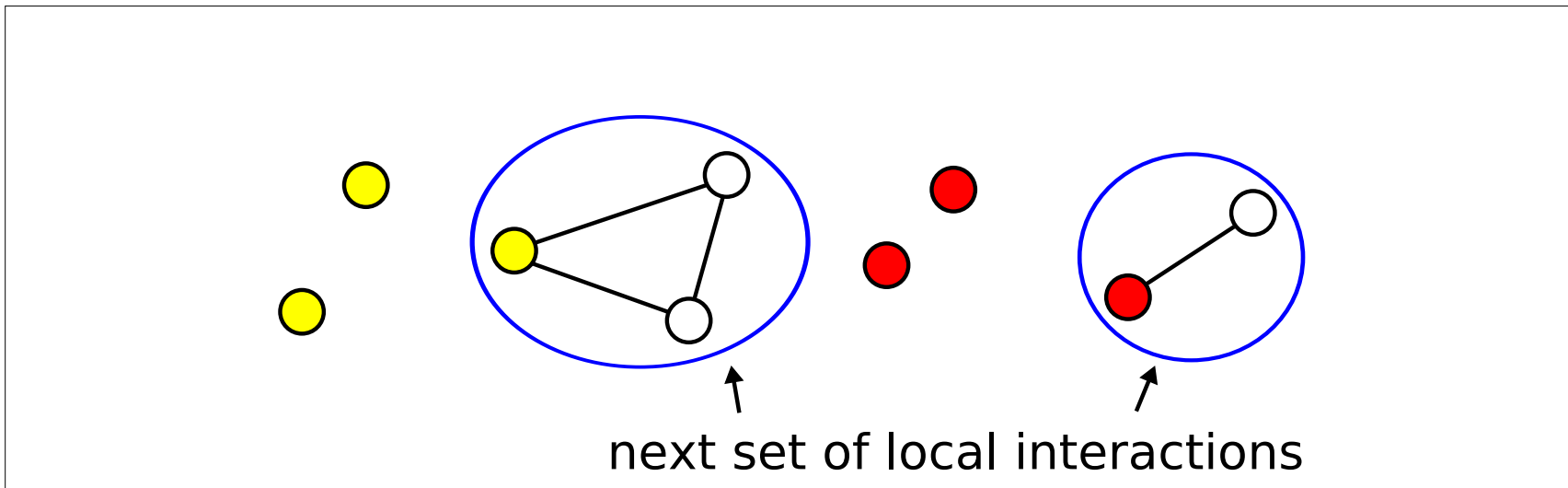
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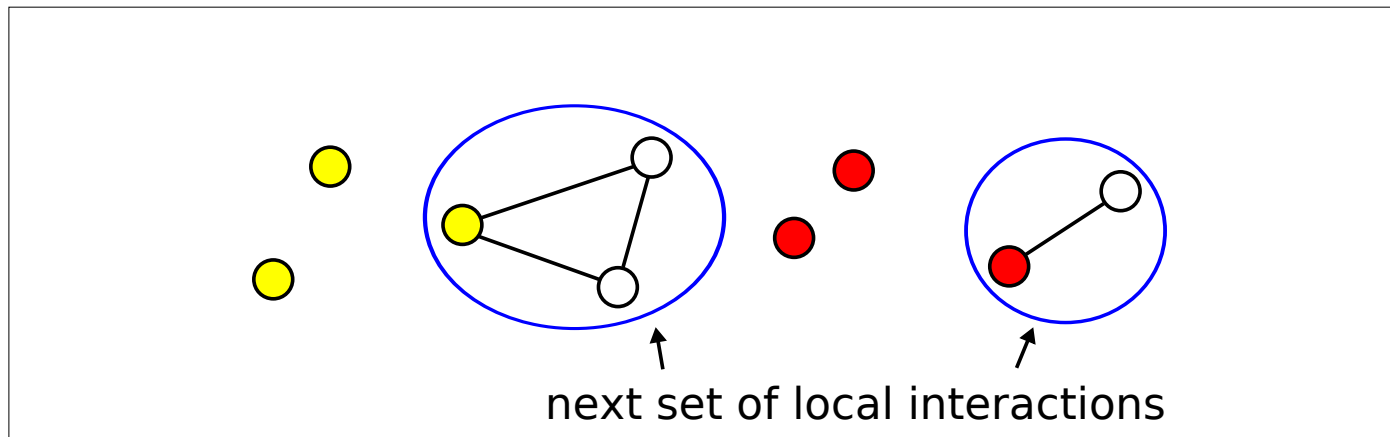
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- Local interactions



- **Goal:** Develop theories and proofs of distributed systems captured by local interactions

# Stages of Refinement



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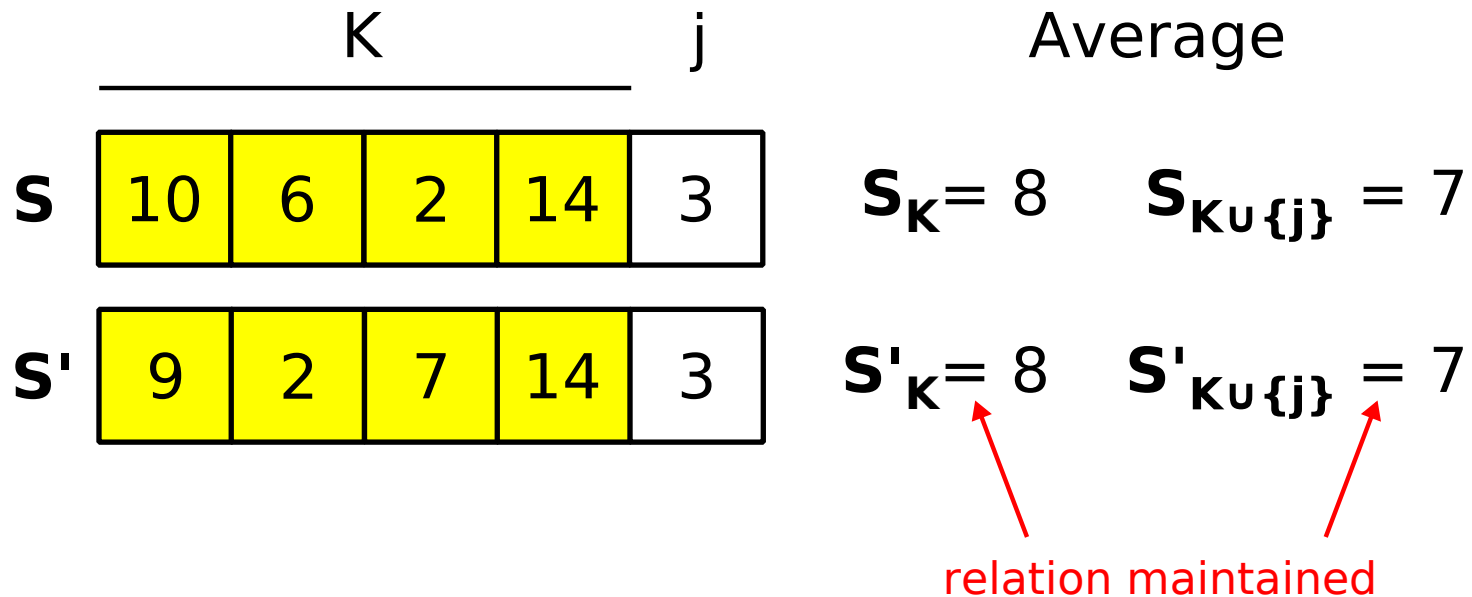


# Local-Global Relations

Relates global state to local interactions

$$\forall j \notin K : S_K \trianglerighteq S'_K \wedge S(j) = S'(j) \implies (S_{K \cup \{j\}}) \trianglerighteq (S'_{K \cup \{j\}})$$

- $\trianglerighteq$  is a transitive binary relation
- $K$  is a nonempty subset of agents



# System Specification

- Agents  $\mathcal{A}$ , each with a value of type  $\mathcal{T}$

agent: TYPE, T: TYPE

- States  $S, S' \in \mathcal{S}$

- $S : \mathcal{A} \rightarrow \mathcal{T}$ , thus  $S(k)$  is agent state

state: TYPE = FUNCTION[agent -> T]

- Let  $f$  be a function over sets of agents

$$f : \mathcal{S} \times \mathcal{A}^+ \rightarrow \mathcal{T}$$

where  $\mathcal{A}^+$  is the power set of  $\mathcal{A}$

f: FUNCTION[state, finite\_set[agent] -> T]

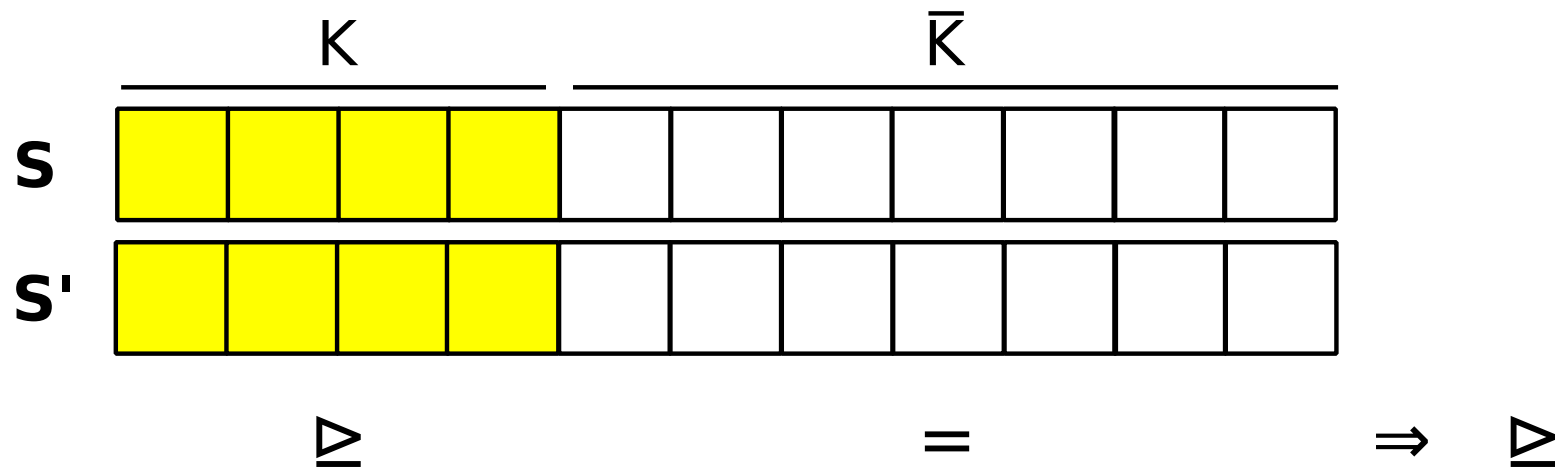
- Notation:  $S_K \equiv f(S, K) \mid K \subseteq \mathcal{A}$

# Generalized Local-Global Relations

- Local-global relations over the entire state

$$S_K \supseteq S'_K \wedge \forall j \notin K : S(j) = S'(j) \implies S_A \supseteq S'_A$$

- $K$  nonempty set of agents



# Generalized Local-Global Relations

- Local-global relations over the entire state

$$S_K \triangleright S'_K \wedge \forall j \notin K : S(j) = S'(j) \implies S_A \triangleright S'_A$$

- $K$  nonempty set of agents

- In PVS

```
1 >: FUNCTION[T, T -> bool]
2
3 lg_relation: LEMMA
4   f(pre, K) > f(post, K) % S_K > S'_K
5   AND (FORALL (j: A | NOT member(j, K)):
6         pre(j) = post(j)) % S(j) = S'(j)
7   IMPLIES
8     f(pre, fullset) > f(post, fullset) % S_A > S'_A
```

# Examples

Minimum

3	4	6	2	8	2
3	4	2	9	4	2

min =

Universal Quantification

T	T	F	F	T	F
T	F	F	T	T	F

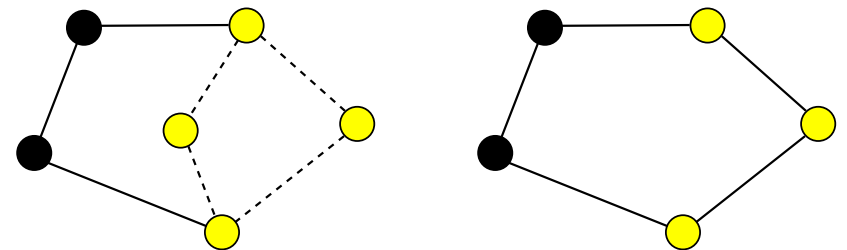
$\wedge =$

Sum of Squares

$\frac{6}{36}$	$\frac{7}{49}$	$\frac{8}{64}$	$\frac{4}{16}$	$\frac{3}{9}$	174
$\frac{6}{36}$	$\frac{7}{49}$	$\frac{5}{25}$	$\frac{2}{4}$	$\frac{3}{9}$	123

$\sum^2 <$

Convex Hull



$C_x \subseteq^+$

# Law of Local-Global Relations

- $S \rightarrow S'$  denotes transition from  $S$  to  $S'$
- Restrict attention to systems where  
 $S \rightarrow S' \implies S_{\mathcal{A}} \trianglerighteq S'_{\mathcal{A}}$
- $\trianglerighteq$  conserved

$$\forall t > 0 : S_{\mathcal{A}}^0 \trianglerighteq S_{\mathcal{A}}^t$$

where  $S^t$  system state after  $t$  transitions

- For example
  - **Conservation** where  $\trianglerighteq$  is =
  - **Non-increasing** where  $\trianglerighteq$  is  $\geq$
  - **Strictly Decreasing** where  $\trianglerighteq$  is  $>$
- Follows from transitivity of  $\trianglerighteq$

# Stages of Refinement



# Local-Global Operator Refinement

- Let  $\circ$  be a binary operator over  $\mathcal{T}$

$$\circ : \mathcal{T} \times \mathcal{T} \rightarrow \mathcal{T}$$

with identity element  $\bar{0}$

$$(a \circ \bar{0} = a) \wedge (\bar{0} \circ a = a)$$

- In PVS

```
1      o: VAR FUNCTION[T, T -> T]
2  zero: VAR T
3
4  identity?(o, zero): bool =
5    FORALL (x: T):
6      x o zero = x AND zero o x = x
```

# Local-Global Operator Refinement

- Define fold

$$\text{fold}(S, K, \circ) = \begin{cases} \bar{0} & \text{if } K = \emptyset, \\ S(k) \circ \text{fold}(S, K \setminus \{k\}, \circ) & \text{otherwise,} \end{cases}$$

where  $k$  is some element in  $K$ .

# Local-Global Operator Refinement

- Define fold

$$\text{fold}(S, K, \circ) = \begin{cases} \bar{0} & \text{if } K = \emptyset, \\ S(k) \circ \text{fold}(S, K \setminus \{k\}, \circ) & \text{otherwise,} \end{cases}$$

where  $k$  is some element in  $K$ .

- In PVS

```
1 fold(S: state,  
2     K: finite_set[agent]): RECURSIVE T =  
3   IF empty?(K) THEN zero  
4   ELSE S(choose(K)) o fold(S, rest(K))  
5   ENDIF  
6 MEASURE card(K)
```

# Local-Global Proof Obligations

- Would like to show that `fold` maintains our local global relation
- Required to prove that the definition is satisfied

$\forall j \notin K :$

$$\text{fold}(S, K, \circ) \triangleright \text{fold}(S', K, \circ) \wedge S(j) = S'(j) \implies \\ \text{fold}(S, K \cup \{j\}, \circ) \triangleright \text{fold}(S', K \cup \{j\}, \circ)$$

# Local-Global in PVS

```
1 local_global[
2   agent: TYPE,
3     T: TYPE,
4     f: FUNCTION[state, finite_set[agent] -> T],
5     >: FUNCTION[T, T -> bool]: THEORY BEGIN
6 ASSUMING
7   R_transitive: ASSUMPTION transitive?(>)
8   f_local_global: ASSUMPTION
9     FORALL (pre, post: state,
10      K: finite_set[agent],
11      k: agent | NOT member(k, K)):
12     f(pre, K) > f(post, K) AND pre(k) = post(k) IMPLIES
13     f(pre, add(k, K)) > f(post, add(k, K))
14 ENDASSUMING
```

# fold **Theory** in PVS

Must discharge assumption on `IMPORT`

```
1 fold[
2   agent: TYPE,
3     T: TYPE,
4     o: FUNCTION[T, T -> T],
5   zero: T,
6     >: FUNCTION[T, T -> bool]
7 ]: THEORY BEGIN
8   fold(S: state, K: finite_set[agent]): T
9
10  IMPORTING local_global[agent, T, fold, >]
11 END fold
```

# TCCs of Local-Global

```
1  % Assuming TCC generated (at line 57, column 12) for
2      % local_global[agent, T, fold, >]
3      % generated from assumption local_global.f_local_global
4      % proved - complete
5  IMP_local_global_TCC1: OBLIGATION
6      FORALL (pre, post: state, K: finite_set[agent]):
7      FORALL (k: agent | NOT member(k, K)):
8          (fold(pre, K) > fold(post, K) AND pre(k) = post(k) IMPLIES
9              fold(pre, add(k, K)) > fold(post, add(k, K)));
```

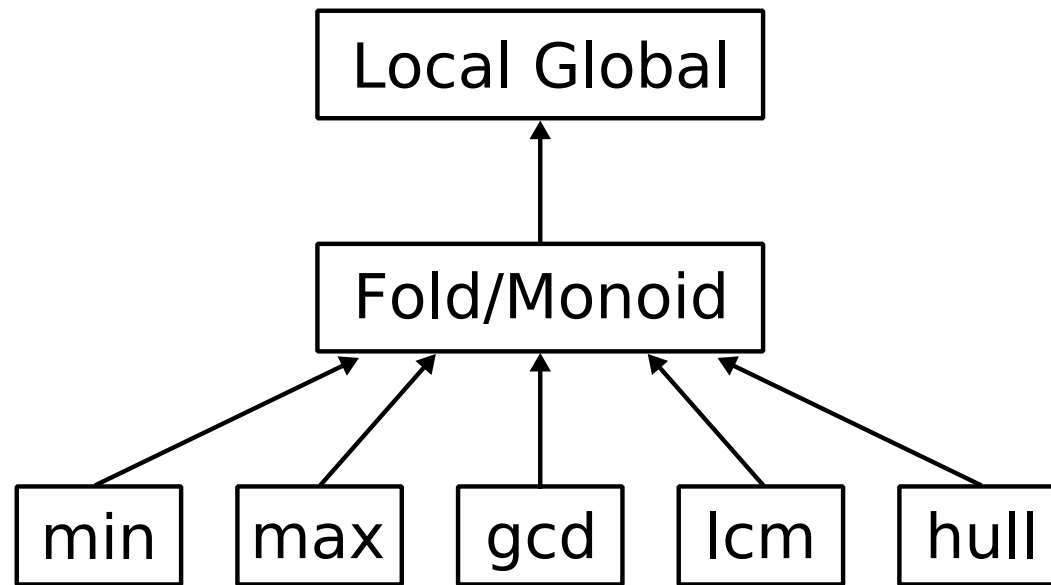
# Proving Local-Global

- If  $(\mathcal{T}, \circ)$  is a commutative monoid with identity element  $\bar{0}$  and  $\circ$  is monotonic, then  $(\text{fold}, \underline{\triangleright})$  is local-global
- A monoid is
  - $(a \circ b) \circ c = a \circ (b \circ c)$
  - $(a \circ \bar{0} = a) \wedge (\bar{0} \circ a = a)$
  - $a, b \in \mathcal{T} \wedge (a \circ b) = c \implies c \in \mathcal{T}$
- Monotonicity is
  - $\forall a, b, c \in T : a \underline{\triangleright} b \implies (a \circ c) \underline{\triangleright} (b \circ c)$

# PVS Assumptions on $\circ$

```
1 fold[agent: TYPE, T: TYPE, o: FUNCTION[T, T -> T],
2   zero: T, >: FUNCTION[T, T -> bool]]: THEORY BEGIN
3   ASSUMING
4     zero_identity: ASSUMPTION identity?(o)(zero)
5     o_associative: ASSUMPTION associative?(o)
6     o_commutative: ASSUMPTION commutative?(o)
7     o_closed:      ASSUMPTION closed?(o)
8     o_monotonic:   ASSUMPTION
9       FORALL (u, v, w: T): u > v IMPLIES u o w > v o w
10  ENDASSUMING
11  fold(S: state, K: finite_set[agent]): T
12  IMPORTING local_global[agent, T, fold, >]
13 END fold
```

# Stages of Refinement

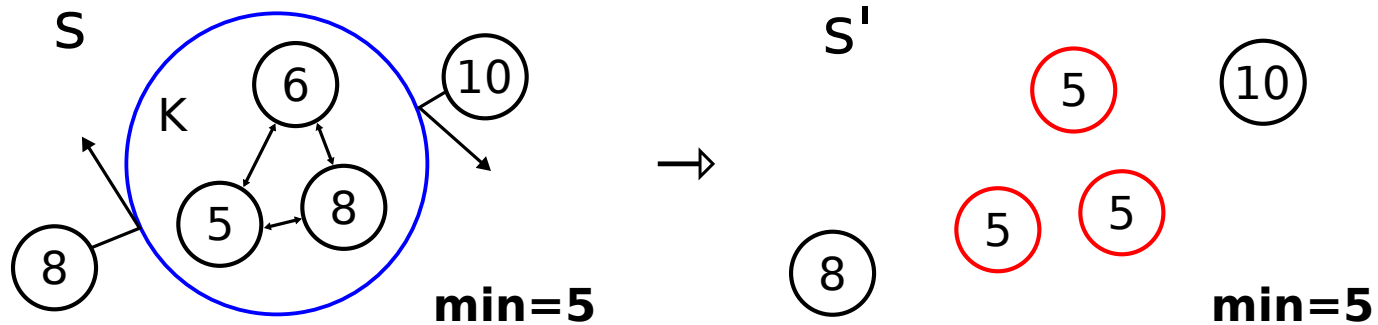


# Example: Consensus

- Given a distributed system with  $n$  agents
- System state is an array  $S$  where  $S(j)$  is the state of agent  $j$
- Initial system state:  $S^0$
- Action:  $\forall k \in K : S(k) = f(S, K)$
- Desired final state:  $S^*$  where  $\forall j \in \mathcal{A} : S^*(j) = f(S^0)$ 
  - Example  $f$ 's:
    - Minimum
    - Maximum
    - Greatest common divisor
    - Least common multiple
    - Convex hull
- Consider generic  $f$ 's: fold,  $\circ$

# Theory Instantiation

Example: min consensus



● **Proof obligation:** operators fit our fold assumption

# In PVS

## ● Importing fold

```
1 min: THEORY
2 BEGIN
3   min(m, n: real): {p: real | p <= m AND p <= n} =
4     IF m > n THEN n ELSE m ENDIF
5   % Recall: fold[agent, T, o, zero, >]
6   IMPORTING fold[posnat, real, min, posinf, >=]
7 END min
```

# In PVS

## ● Importing `fold`

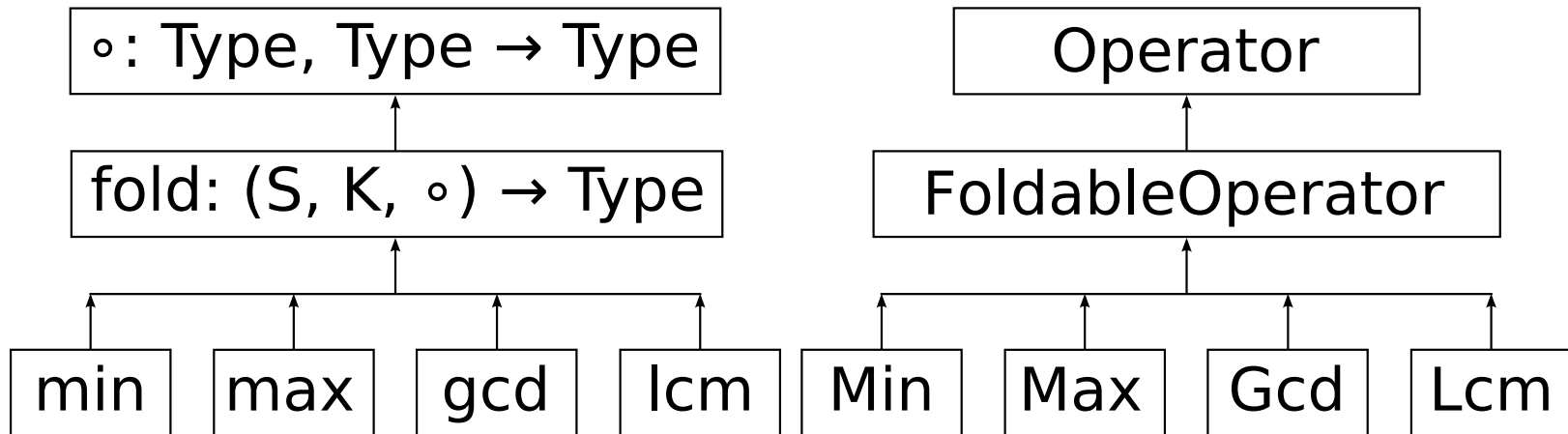
```
1 min: THEORY
2 BEGIN
3   min(m, n: real): {p: real | p <= m AND p <= n} =
4     IF m > n THEN n ELSE m ENDIF
5   % Recall: fold[agent, T, o, zero, >]
6   IMPORTING fold[posnat, real, min, posinf, >=]
7 END min
```

## ● Enforces `o` assumptions (e.g. monotonicity):

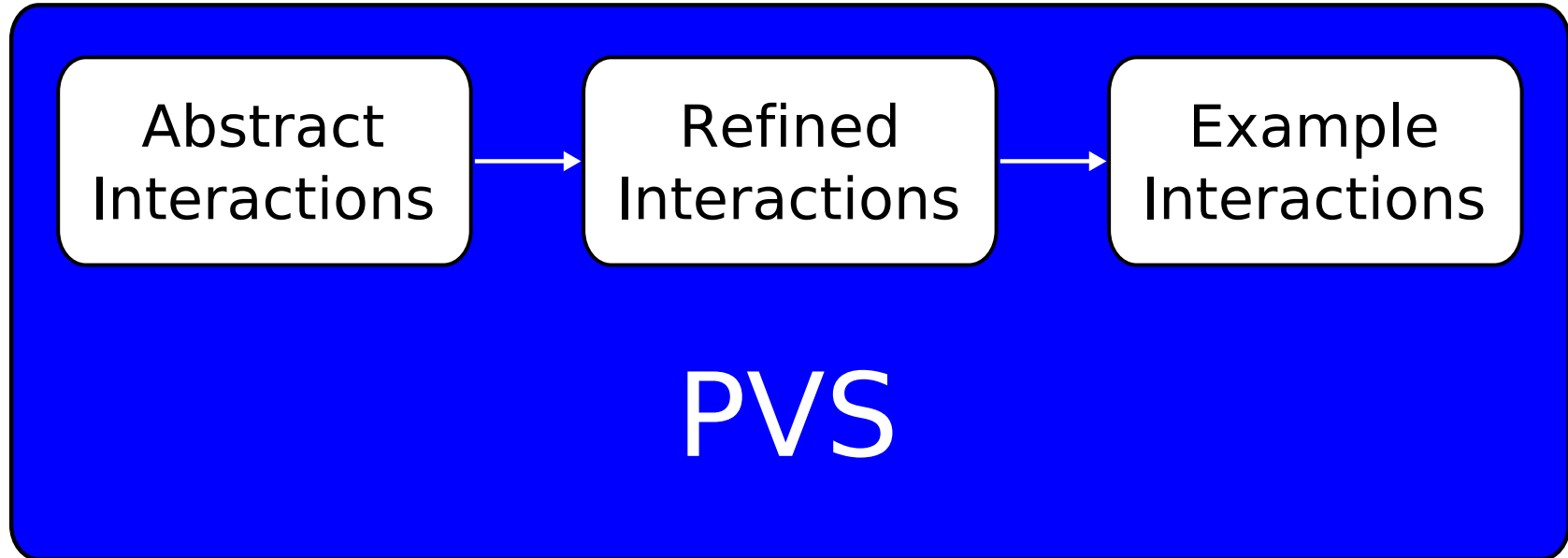
```
1 % Assuming TCC generated (at line 10, column 12) for
2   % fold[posnat, real, min, posinf, >=]
3   % generated from assumption fold.o_monotonic
4 IMP_fold_TCC5: OBLIGATION
5   FORALL (u, v, w: real): u >= v IMPLIES
6     min(u, w) >= min(v, w);
```

# Conclusion

- Steep learning curve
- Forces a focus on structure and proof details
- Modularity is rewarded
  - efficient for proving
  - efficient for implementing
- Theory hierarchy maps to Java Object hierarchy



# Stages of Refinement



<http://www.infospheres.caltech.edu/muri2009>

# Appendix

# Install and Run PVS

- Download from <http://pvs.csl.sri.com/download.shtml>
- Latest (4.2) `pvs-4.2-ix86-Linux-allegro.tgz`
- From shell run `$PVS/bin/relocate` to set path
- Run `$PVS/pvs`
- Overview of the commands: `M-x pvs-help` (`C-h p`)
- `M-x exit-pvs` (`C-x C-c`)

# Applications

- Examples
  - Hardware verification
  - Sequential and Distributed algorithms verification
  - Critical real-time systems verification

# PVS Types

- Base and Build-in types
  - `bool, int, real, nat, ...`
- User-defined types:
  - Keyword: `TYPE` or `TYPE+` (non empty)
- Uninterpreted Type
  - `Type1 : TYPE`
  - Defined equality predicate. Given two elements, whether they are the same or not
- Subtype
  - `Type2 : TYPE = { x : nat | x > 0 }`
  - `Type3 (n : int) : TYPE = { i : nat | i >= n }`

# PVS Types

## • Enumeration Type

- $\text{Type4:TYPE} = \{\text{Type1}, \text{Type2}\}$

## • Function Type

- $\text{Type5:TYPE} = [\text{int} \rightarrow \text{int}]$

- $\text{Type6:TYPE} = \text{FUNCTION} [\text{int} \rightarrow \text{int}]$

- $\text{Type7:TYPE} = \text{ARRAY} [\text{int} \rightarrow \text{int}]$

- $\text{Type5}, \text{Type6}, \text{Type7}$  are equivalent

- $\text{Type8:TYPE} = [\text{int}, \text{int} \rightarrow \text{int}]$

## • Record Types

- $\text{Type9:TYPE} = [\# \text{t1:Type1}, \text{t2:Type2} \#]$

# Declarations

- Variable Declarations

- `n, m, p: VAR nat`

- Constant Declarations

- `k: nat`

- `sum(i, j: nat): nat`

- `k: nat = 10`

- `next(n): int = n+1`

- `Less_than_10?(m): bool = m < 10`

- `fact(n): RECURSIVE nat =`

- `IF n=0 THEN 1 ELSE n*fact(n-1) ENDIF`

- `MEASURE n`

# Declarations

- Formula Declarations

- transitive:

- AXIOM  $n < m$  AND  $m < p$  IMPLIES  $n < p$

- Others: CLAIM, FACT, LEMMA, PROPOSITION, THEOREM, ...

# Expressions

## ● Boolean

- =, /= TRUE, FALSE, AND, OR, IMPLIES, IFF

## ● If-then-else

- IF cond THEN exp1 ELSE exp2 ENDIF

## ● Numeric

- 0, 1, ..., +, \*, /, -, <, >, ...

## ● Binding (local scope for variables)

- FORALL, EXISTS

## ● Records

- `l:list = (# node:=val1, nxt:=val2 #)`
- Accessors: `l`node`, `node(l)`, `l`nxt`, `nxt(l)`
- Update: `l WITH [node:=val3, nxt:= val4]`

# Some Rules

## ● Propositional Rules

- `flatten`: disjunctive simplification
- `case`: case splitting
- `prop`: propositional simplification

## ● Quantifier Rules

- `skolem`: skolemize a universally quantified variable
- `inst`: instantiate an existentially quantified variable

## ● Induction rules

- `induct`: invoke induction scheme

# Some Rules

- Rules for using definitions and lemmas
  - `expand`: expanding a function or type definition
  - `lemma`: introduce the statement of a lemma as an assumption
- Rules for simplification
  - `assert`, `bddsimp`: **simplify**
  - `smash`, `grind`: **lift-it, rewrite, and repeatedly simplify**
- Control
  - `quit`, `postpone`, `undo`

# Strategies and Automation

- User-defined strategies: Saved in pvs-strategies

- `(DEFSTEP strategy-name (parameters)  
strategy-expression  
documentation-string format-string )`

- `(try step1 step2 step3)`

Applies step1 to the current goal. If step1 succeeds and generate sub-goals, then step2 is applied; otherwise step3 is applied to the current goal.

- `(repeat step1)`

- Examples

- `(ground)`

- `(try (flatten) (propax) (split))`

- `(try (try (flatten) (fail) (skolem 1 ("a" "b")))  
(postpone))`